# L航基础数学青年论坛

2018年,11月30日上午九点--12月1日下午六点 北航旧主楼321室,我们欢迎您的参与!

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Nov 30 Dec 01



主办单位: 北京航空航天大学

基金支持 、计划项目 国家青年千

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邀请报告



### 2018北航基础数学青年论坛会议报告日程

#### 9:00-9:30 开幕式及照相 报告人 报告题目 主持人 时间 Fargues-Rapoport conjecture for some 陈苗芬 文志英 9:30-10:15 华东师范大学 p-adic period domains 茶歇(旧主楼314) 10:15-10:35 RC-positivity and Yau's rational 杨晓奎 10:35-11:20 中科院数学院 conectedness conjecture 梁湘玉 于品 11:20-12:05 从几何的观点看波动方程 清华大学 2018年11月30日下午 14:00-17:30,北航旧主楼321 报告人 时间 报告题目 主持人 On deformational spectral rigidity of 魏巧玲 14:00-14:45 首都师范大学 convex planar domains 周渊 方扬钦 14:45-15:30 Plateau问题 华中科技大学 茶歇(旧主楼314) 15:30-16:00 Mod \$p\$ cohomology of Shimura 胡永泉 16:00-16:45 中科院数学院 curves (I) 唐舜 Mod \$p\$ cohomology of Shimura 王浩然 16:45-17:30 清华大学 curves (II) 2018年11月1日上午9:00-11:45,北航旧主楼321 时间 报告人 报告题目 主持人 鄢敬之 9:00-9:45 About Poincare-Birkhoff type theorems 薛玉梅 四川大学 茶歇(旧主楼205) 9:45-10:15 Galois action on representations of 童纪龙 10:15-11:00 首都师范大学 fundamental groups 陈苗芬 Severi type inequalities and binational 江智 11:00-11:45 复旦大学 geometry of irregular threefolds 2018年12月1日下午 14:00-17:30, 北航旧主楼321 报告人 报告题目 时间 主持人 Crossing Probabilities in 2D Critical 吴昊 14:00-14:45 清华大学 Lattice Models 林勇 On Bifurcation of Eigenvalues Along 常寅山 14:45-15:30 四川大学 **Convex Symplectic Paths** 茶歇(旧主楼205) 15:30-16:00 On the Rost kernel of division algebras 胡勇 16:00-16:45 南方科技大学 over discrete valuation fields 高鹏 吴涵 16:45-17:30 On Kuznetsov-Bykovskii's formula 洛桑联邦理工

#### 2018年11月30日上午 9:00-12:05, 北航旧主楼321

## 常寅山: On Bifurcation of Eigenvalues Along Convex Symplectic Paths

We consider a continuously differentiable curve  $t \mapsto \gamma(t)$  in the space of  $2n \times 2n$  real symplectic matrices, which is the solution of the following ODE:

$$\frac{\mathrm{d}\gamma}{\mathrm{d}t}(t) = J_{2n}A(t)\gamma(t), \gamma(0) \in \mathrm{Sp}(2n,\mathbb{R}),$$

where  $J = J_{2n} \stackrel{\text{def}}{=} \begin{bmatrix} 0 & \text{Id}_n \\ -\text{Id}_n & 0 \end{bmatrix}$  and  $A: t \mapsto A(t)$  is a continuous path in the space of  $2n \times 2n$  real matrices which are symmetric. Under a certain convexity assumption (which includes the particular case that A(t) is strictly positive definite for all  $t \in \mathbb{R}$ ), we investigate the dynamics of the eigenvalues of  $\gamma(t)$  when t varies, which are closely related to the stability of such Hamiltonian dynamical systems. We rigorously prove the qualitative behavior of the branching of eigenvalues and explicitly give the first order asymptotics of the eigenvalues. This generalizes classical Krein-Lyubarskii theorem on the analytic bifurcation of the Floquet multipliers under a linear perturbation of the Hamiltonian.

## 陈苗芬: Fargues-Rapoport conjecture for some p-adic period domains

Rapoport and Zink introduce the p-adic period domain (also called the admissible locus) and the weakly admissible locus inside the rigid analytic p-adic flag varieties. Over the admissible locus, there exists a universal crystalline Qp-local system which interpolates a family of crystalline representations attached to all classical points. The weakly admissible locus is an approximation of the admissible locus obtained by removing a profinite set of closed Schubert varieties. In this talk, we will prove Fargues-Rapoport conjecture for the basic local Shimura datum which gives a group theoretic characterization when the admissible locus and the weakly admissible locus coincide. If time permits, we will also discuss about the generalization to the non-basic case. Part of this talk is based on a joint work with Laurent Fargues and Xu Shen.

#### 方扬钦: Plateau 问题

Plateau 问题起源于 18 世纪,是一类给定边界的极小"曲面"的存在性问题。直到上世纪 才有重大突破,几何测度论也由此孕育而生。到目前为止,仍有众多问题未得到解决。 我们今天将探讨 Plateau 问题的存在性以及解的正则性。

### 胡勇: On the Rost kernel of division algebras over discrete valuation fields

Let F be a field,  $\ell$  a prime and D a central division F-algebra of period  $\ell$ . By the Rost kernel of D we mean the subgroup of  $F^*$  consisting of elements  $\lambda$  such that the cohomology class  $(D) \cup (\lambda) \in H^3(F, \mathbb{Q}_{\ell}/\mathbb{Z}_{\ell}(2))$  vanishes. In general, this subgroup contains products of reduced norms from D and  $\ell$ -th powers in  $F^*$ . Suslin conjectured that there are no other elements in the Rost kernel. In this talk, I will discuss Suslin's conjecture for complete discrete valuation fields with nice residue fields. This is a joint work with Wu Zhengyao.

#### 胡永泉: Mod p cohomology of Shimura curves (I)

At present, the mod p (and p-adic) local Langlands correspondence is only well understood for the group  $\operatorname{GL}_2(\mathbb{Q}_p)$ . One of the main difficulties is that little is known about supersingular representations besides this case, and we do know that there is no simple one-to-one correspondence between representations of  $\operatorname{GL}_2(L)$  with two-dimensional representations of  $\operatorname{Gal}(\overline{L}/L)$ , at least when  $L/\mathbb{Q}_p$  is (non-trivial) finite unramified. However, the Buzzard-Diamond-Jarvis conjecture and the mod plocal-global compatibility for  $\operatorname{GL}_2/\mathbb{Q}$  suggest that this hypothetical correspondence may be realized in the cohomology of Shimura curves with characteristic p coefficients (cut out by some modular residual global representation  $\overline{r}$ ). In a joint work in progress we obtain more information about this correspondence.

In the first talk (Y. H.), we will explain the background on representation theory of  $GL_2$  and state the main result. In the second talk (H. W.), we will explain some key ingredients in the proof.

## 江智: Severi type inequalities and binational geometry of irregular threefolds

We will explain how to deduce a Severi type inequality on general irregular variety with the help of cohomological rank functions. We will also explain some results on the birational geometry of irregular threefolds as an application of such inequality.

## 童纪龙: Galois action on representations of fundamental groups

In this talk, we shall investigate the Galois action in the category of representations of fundamental groups in the realm of p-adic Simpson correspondance. We shall mainly focus in the finite field case. This is a join work in progress with Sheng Mao.

#### 王浩然: Mod p cohomology of Shimura curves (II)

At present, the mod p (and p-adic) local Langlands correspondence is only well understood for the group  $\operatorname{GL}_2(\mathbb{Q}_p)$ . One of the main difficulties is that little is known about supersingular representations besides this case, and we do know that there is no simple one-to-one correspondence between representations of  $\operatorname{GL}_2(L)$  with two-dimensional representations of  $\operatorname{Gal}(\overline{L}/L)$ , at least when  $L/\mathbb{Q}_p$  is (non-trivial) finite unramified. However, the Buzzard-Diamond-Jarvis conjecture and the mod plocal-global compatibility for  $\operatorname{GL}_2/\mathbb{Q}$  suggest that this hypothetical correspondence may be realized in the cohomology of Shimura curves with characteristic p coefficients (cut out by some modular residual global representation  $\overline{r}$ ). In a joint work in progress we obtain more information about this correspondence.

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## 魏巧玲: On deformational spectral rigidity of convex planar domains

This talk is related to the famous question "Can you hear the shape of drum?" by M.Kac in 1960'. That is, whether a planar domain  $\Omega$  can be uniquely determined by its Laplace spectrum consisting of eigenvalues of a Dirichlet problem . In general, there are counterexamples.

Meanwhile, from dynamical aspect, there is length spectrum consisting of perimeters of all periodic orbits of a billiard problem inside  $\Omega$ . The Laplace and length spectra are closely related, generically the first determines the second.

During the talk we show that a planar axis symmetric domain close to the circle can not be smoothly deformed preserving the length spectrum unless the deformation is a rigid motion. This is a joint work with J. De Simoi and V.Kaloshin.

#### 吴涵: On Kuznetsov-Bykovskii's formula

In 2013, Soundararajan and Young improved the classical bounds of the error term in the prime geodesic theorem for the full modular group  $PSL_2(\mathbb{Z})$  due to Iwaniec and Sarnak. This improvement was recently (2018) generalized to the Picard manifold for  $PSL_2(\mathbb{Z}[i])$  by Balkanova-Chatzabos-Cherubini-Frolenkov-Laaksonen and Balkanova-Frolenkov. As a main new input with respect to the proofs of the classical bounds, a formula due to Kuznetsov and presented by Bykovskii, which relates the counting functions  $\Psi_{\Gamma}$  of the partial sums of the analogue of von-Mangoldt functions with the partial sums of certain *L*-series related to quadratic Dirichlet characters, plays an important role. The existing proofs of this formula are "arithmetic" in nature and seem to be difficult to generalize to congruence subgroups. In this talk, we will give an analytic proof of this formula, which works well in the number field setting and for all congruence subgroups. In particular, we will emphasize its intimate relation with some Rankin-Selberg unfolding.

#### 吴昊: Crossing Probabilities in 2D Critical Lattice Models

The planar Ising model is one of the most studied lattice models in statistical physics. It was introduced in the 1920s by W. Lenz as a model for magnetic materials. R. Peierls showed in 1936, in two (and higher) dimensions, an order-disorder phase transition in fact occurs at a certain critical temperature. Ever since, there has been active research to understand the 2D Ising model at criticality, where it enjoys conformal invariance in the scaling limit. In this talk, we give crossing probabilities of multiple interfaces in the critical planar Ising model with alternating boundary conditions. Besides, we also explain that a similar formula on the crossing probabilities also holds for critical Percolation and level lines of Gaussian Free Field.

#### 鄢敬之: About Poincare-Birkhoff type theorems

In this talk, I would like to introduce several Poincare-Birkhoff type theorems, some generalizations and an application to local dynamics of surface homeomorphisms.

### 杨晓奎: RC-positivity and Yau's rational conectedness conjecture

In this presentation, we will describe the relationship between various positivity notions in differential geometry and algebraic geometry. We shall also introduce a new concept called "RC-positivity" in differential geometry and use it to characterize uniruled and rationally connected projective manifolds. In particular, we confirm a conjecture of Yau that a compact Kahler manifold with positive holomorphic sectional curvature is projective algebraic and rationally connected.

### 于品:从几何的观点看波动方程

波的传播是自然界中最普通的现象,也是众多经典物理现象中物质演化和能量得以重新 分配的基本机制,比如说电磁波、引力波或者可压缩流体中的某些激波。数学上,这些 非线性波的传播通常可以用偏微分方程来刻画。这些方程差别很大,对应的物理现象也 很不一样。然而,我们将解释它们背后都对应着统一的几何图象并且尝试解释如何用这 种几何背景来研究相应的偏微分方程。